
Math 3360 - Foundations of Algebra I: Problem Sheet 1

- In each of the following cases, sketch an example of a function $f : \mathbb{R} \rightarrow \mathbb{R}$ such that
 - f is surjective but not injective.
 - f is injective but not surjective.
 - f is both surjective and injective.
 - f is neither surjective nor injective.
- Determine whether the following functions are one-to-one and/or onto. If the function is invertible find its inverse.
 - $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 2x + 5$.
 - $f : \mathbb{Q}^+ \rightarrow \mathbb{Q}^+$ given by $f(x) = \frac{1}{x}$.
 - $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2 - 4$.
 - $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = |x + 1|$.
- Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions. Prove that if both f and g are invertible then $g \circ f$ is invertible with $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(Note: do not assume that $g \circ f$ is one-to-one or onto.)